

ECE 109: Comprehensive Equation Reference

Every Equation with Complete Variable Definitions (Lessons 1–6)

Physical Constants

h	$6.626 \times 10^{-34} \text{ J s}$
$\hbar = h/2\pi$	$1.055 \times 10^{-34} \text{ J s}$
hc	1240 eV nm
m_e	$9.109 \times 10^{-31} \text{ kg}$
e	$1.602 \times 10^{-19} \text{ C}$
k_B	$1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$
ϵ_0	$8.854 \times 10^{-12} \text{ F/m}$
μ_0	$4\pi \times 10^{-7} \text{ N/A}^2$
c_0	$3.0 \times 10^8 \text{ m/s}$
N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$
a_0	$0.0529 \text{ nm (Bohr radius)}$
$e^2/(4\pi\epsilon_0)$	$1.44 \text{ eV nm (Coulomb shortcut)}$
$k_B T _{300\text{K}}$	0.02585 eV

h = Planck's constant; $\hbar$ = reduced Planck's constant ("h-bar"); m_e = electron rest mass; e = elementary charge (magnitude of electron charge); k_B = Boltzmann constant; ϵ_0 = permittivity of free space; μ_0 = permeability of free space; c_0 = speed of light in vacuum; N_A = Avogadro's number (atoms per mole); a_0 = Bohr radius (most probable electron–nucleus distance in hydrogen ground state). Constants are referenced by symbol throughout this document.

Wave–Particle Duality

$$E = hf = \hbar\omega$$

Planck–Einstein relation: energy of a single photon. E = photon energy [J or eV]; h = Planck's constant; f = frequency of the electromagnetic radiation [Hz]; $\hbar = h/(2\pi)$; $\omega = 2\pi f$ = angular frequency [rad/s].

$$p = \frac{h}{\lambda} = \frac{hf}{c}$$

Photon momentum. p = momentum of the photon [kg m/s]; λ = wavelength of the radiation [m]; c = speed of light in the medium [m/s].

$$\text{Photoelectric: } KE_{\max} = hf - \Phi$$

Photoelectric effect: maximum kinetic energy of electrons ejected from a surface by incident light. $KE_{\max}$ = maximum kinetic energy of emitted photoelectrons [J or eV]; hf = energy of the incident photon; Φ = work function of the material, the minimum energy required to extract an electron from the surface [eV]. No emission occurs if $hf < \Phi$.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV}}$$

de Broglie wavelength: every particle with momentum p has an associated wavelength. λ = de Broglie wavelength [m]; p = particle momentum [kg m/s]; m_e = electron mass; e = elementary charge [C]; V = accelerating potential difference the electron has been accelerated through [V]. The second form applies to an electron accelerated from rest through potential V , using $KE = eV = p^2/(2m_e)$.

$$\text{Wien } \lambda_{\max} T = 2.898 \times 10^{-3} \text{ m K}$$

Wien's displacement law: relates the peak emission wavelength of a blackbody to its temperature. $\lambda_{\max}$ = wavelength at which spectral radiance is maximum [m]; T = absolute temperature of the blackbody [K]. Hotter objects emit at shorter wavelengths.

$$\text{Photon flux } \dot{N} = \frac{P}{E_{ph}} = \frac{P\lambda}{hc}$$

Rate of photon emission from a monochromatic source. $\dot{N}$ = number of photons emitted per second [photons/s]; P = total radiated power of

the source [W]; $E_{ph} = hf = hc/\lambda$ = energy of a single photon [J]; λ = wavelength [m].

$$\text{Thermal } \lambda_B = h/\sqrt{3mk_B T}$$

Thermal de Broglie wavelength. λ_B = de Broglie wavelength of a particle at thermal equilibrium [m]; m = particle mass [kg]; k_B = Boltzmann constant; T = absolute temperature [K]. Quantum effects become important when λ_B approaches the average interparticle spacing.

Schrödinger Equation

$$\text{Momentum op. } \hat{p} = -i\hbar \frac{\partial}{\partial x}$$

Quantum mechanical momentum operator (1D). $\hat{p}$ = momentum operator; $i = \sqrt{-1}$ = imaginary unit; $\hbar$ = reduced Planck's constant; $\partial/\partial x$ = partial derivative with respect to position x [m]. Applied to $\psi(x)$ to extract momentum information.

$$\text{Expectation } \langle A \rangle = \int \psi^* \hat{A} \psi dx$$

Expectation value of an observable. $\langle A \rangle$ = average value of observable A over many measurements; ψ^* = complex conjugate of the wavefunction ψ ; $\hat{A}$ = quantum operator corresponding to observable A (e.g., $\hat{p}$ for momentum, $\hat{H}$ for energy); ψ = wavefunction of the particle; dx = integration over all space.

$$\text{Normalization } \int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

Born interpretation: the particle must be found somewhere. $|\psi(x)|^2$ = probability density, the probability of finding the particle per unit length at position x [m⁻¹]; the integral of this over all space equals 1.

$$\text{TDSE } i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$$

Time-dependent Schrödinger equation: governs how a quantum state evolves in time. i = imaginary unit; $\hbar$ = reduced Planck's constant; $\partial/\partial t$ = partial time derivative; $|\psi\rangle$ = quantum state (wavefunction); $\hat{H}$ = Hamiltonian operator (total energy operator).

$$\text{Hamiltonian } \hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r})$$

Hamiltonian: total energy operator = kinetic + potential. $\hat{H}$ = Hamiltonian operator; $\hbar$ = reduced Planck's constant; m = particle mass [kg]; $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ = Laplacian operator (3D second spatial derivative); $V(\mathbf{r})$ = potential energy as a function of position [J].

$$\text{TISE } \frac{d^2\psi}{dx^2} + \frac{2m_e}{\hbar^2} (E - V)\psi = 0$$

Time-independent Schrödinger equation (1D). $\psi(x)$ = spatial part of the wavefunction (depends only on position); x = position [m]; m_e = electron mass; $\hbar$ = reduced Planck's constant; E = total energy of the particle [J or eV]; $V = V(x)$ = potential energy at position x [J or eV]. Solutions give the stationary (time-independent) states.

Infinite Potential Well

Particle confined to $0 < x < a$ by infinitely high walls: $V = 0$ inside, $V = \infty$ outside.

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

Normalized wavefunctions. $\psi_n(x)$ = wavefunction for the n th quantum state; a = width of the well [m]; $n = 1, 2, 3, \dots$ = quantum number (positive integer; $n = 1$ is the ground state); x = position within the well [m]. The $\sqrt{2/a}$ prefactor ensures normalization.

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2m_e a^2} = \frac{n^2 h^2}{8m_e a^2}$$

Quantized energy levels. E_n = energy of the n th state [J or eV]; n = quantum number; $\hbar$ = reduced Planck's constant; h = Planck's constant; m_e = electron mass; a = well width [m]. Both forms are equivalent. Energy scales as n^2 and as $1/a^2$.

$$|\psi_n|^2 = \frac{2}{a} \sin^2\left(\frac{n\pi x}{a}\right)$$

Probability density for the n th state. $|\psi_n(x)|^2$ = probability per unit length of finding the particle at position x [m⁻¹]. Maxima (antinodes) occur at positions where $\sin^2$ peaks; state n has n antinodes.

$$\Delta E_n = E_{n+1} - E_n = \frac{(2n+1)\hbar^2}{8m_e a^2}$$

Energy spacing between adjacent levels. ΔE_n = energy gap between states n and $n+1$ [J or eV]. Spacing increases with n and decreases with well width a .

$$\text{3D box } E_{n_1 n_2 n_3} = \frac{\hbar^2}{8m_e} \left(\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2} + \frac{n_3^2}{c^2} \right)$$

Energy levels for a particle in a 3D rectangular box. $n_1, n_2, n_3 = 1, 2, 3, \dots$ = independent quantum numbers for each spatial dimension; a, b, c = box dimensions along x, y, z respectively [m]. When $a = b = c$ (cubic box), different (n_1, n_2, n_3) combinations can yield the same energy (degeneracy).

Finite Potential Well

$$k^2 = \frac{2m_e E}{\hbar^2}, \quad \alpha^2 = \frac{2m_e (V_0 - E)}{\hbar^2}$$

Wavevector and decay parameters. k = wavevector inside the well where $E > 0$ [m⁻¹], determines the oscillatory behavior of ψ inside; α = decay constant outside the well where $E < V_0$ [m⁻¹], determines how fast ψ decays into the barrier; m_e = electron mass; E = particle's total energy [J or eV]; V_0 = depth of the potential well (barrier height) [J or eV]; $\hbar$ = reduced Planck's constant.

$$\text{Even } \alpha = k \tan\left(\frac{ka}{2}\right)$$

$$\text{Odd } \alpha = -k \cot\left(\frac{ka}{2}\right)$$

Transcendental equations for bound state energies. “Even” and “Odd” refer to the parity (symmetry) of the wavefunction about the well center. α = decay constant outside; k = wavevector inside; a = well width [m]. These equations have no closed-form solution; each graphical intersection of left and right sides gives one bound state energy.

$$\text{Penetration } \delta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m_e(V_0 - E)}}$$

Penetration depth. δ = characteristic distance the wavefunction extends into the classically forbidden region outside the well [m]; α = decay constant [m⁻¹]; V_0 = barrier height [J or eV]; E = particle energy [J or eV]. The probability density decays as $e^{-2x/\delta}$ outside the well.

$$\text{Bound states } N \leq \left\lfloor \frac{1}{2} + \frac{a}{\pi} \sqrt{\frac{2m_e V_0}{\hbar^2}} \right\rfloor$$

Maximum number of bound states. N = number of bound states the well can support; $\lfloor \cdot \rfloor$ = floor function (round down); a = well width [m]; m_e = electron mass; V_0 = well depth [J or eV]; $\hbar$ = reduced Planck's constant. A finite well always has at least one bound state ($N \geq 1$).

Quantum Tunneling

$$T = \frac{1}{1 + D \sinh^2(\alpha a)}$$

$$D = \frac{V_0^2}{4E(V_0 - E)}, \quad \alpha = \frac{\sqrt{2m_e(V_0 - E)}}{\hbar}$$

Exact transmission coefficient through a rectangular potential barrier. T = transmission probability (dimensionless, $0 \leq T \leq 1$), the probability that a particle incident on the barrier passes through it; D = dimensionless barrier shape parameter; V_0 = barrier height [J or eV]; E = kinetic energy of the incident particle [J or eV], with $E < V_0$; α = decay constant inside the barrier [m⁻¹]; a = barrier width (thickness) [m]; m_e = electron mass; $\hbar$ = reduced Planck's constant; $\sinh(x) = (e^x - e^{-x})/2$ = hyperbolic sine.

$$\text{Thick barrier } T \approx T_0 e^{-2\alpha a}, \quad T_0 = \frac{16E(V_0 - E)}{V_0^2}$$

Approximate transmission for an opaque barrier ($\alpha a \gg 1$). T_0 = pre-exponential factor (dimensionless); the exponential $e^{-2\alpha a}$ dominates, making T extremely sensitive to barrier width a and the energy deficit $V_0 - E$.

$$\text{STM current } I \propto V_b e^{-2\alpha d}, \quad \alpha = \frac{\sqrt{2m_e \phi}}{\hbar}$$

Scanning tunneling microscope. I = tunneling current between tip and sample [A]; V_b = bias voltage applied across the junction [V]; d = tip-to-sample gap distance [m]; ϕ = average work function of tip and sample [eV]; α = decay constant [m⁻¹]; m_e = electron mass; $\hbar$ = reduced Planck's constant. Current is exponentially sensitive to d , enabling sub-angstrom height resolution.

Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}, \quad \Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

Heisenberg uncertainty relations: fundamental quantum limits on simultaneous knowledge of conjugate variables. Δx = standard deviation (uncertainty) of position measurements [m]; Δp = standard deviation of momentum measurements [kg m/s]; ΔE = uncertainty in energy [J or eV]; Δt = uncertainty in time, or the duration over which the energy is measured [s]; $\hbar/2$ = minimum allowed product. These are intrinsic limits of nature, not limitations of measurement apparatus.

Hydrogen Atom

$$V(r) = \frac{-Ze^2}{4\pi\epsilon_0 r}$$

Coulomb potential energy between the nucleus and electron. $V(r)$ = potential energy as a function of distance [J or eV]; Z = atomic number (number of protons in the nucleus; $Z = 1$ for hydrogen); e = elementary charge; ϵ_0 = permittivity of free space; r = radial distance between electron and nucleus [m]. Negative sign indicates electrostatic attraction.

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

Quantized energy levels of the hydrogen atom. E_n = energy of the n th level [eV]; $n = 1, 2, 3, \dots$ = principal quantum number. $n = 1$ is the ground state ($E_1 = -13.6$ eV); $n = \infty$ corresponds to ionization ($E = 0$). Negative energies indicate bound states.

$$L = \hbar\sqrt{\ell(\ell+1)}, \quad L_z = m_\ell \hbar$$

Orbital angular momentum. L = magnitude of the electron's orbital angular momentum [J s or kg m²/s]; $\ell = 0, 1, 2, \dots, n-1$ = orbital (azimuthal) angular momentum quantum number; L_z = component of angular momentum along the z -axis (the measurement axis); $m_\ell = -\ell, -\ell+1, \dots, 0, \dots, +\ell$ = magnetic quantum number ($2\ell+1$ values).

$$S = \hbar\sqrt{s(s+1)} = \frac{\sqrt{3}}{2}\hbar, \quad S_z = m_s \hbar$$

Electron spin angular momentum. S = magnitude of the intrinsic spin angular momentum [J s]; $s = 1/2$ for all electrons (fixed); S_z = z -component of spin; $m_s = +1/2$ (spin-up) or $m_s = -1/2$ (spin-down) = spin magnetic quantum number.

Quantum numbers: $n = 1, 2, \dots$; $\ell = 0 \dots n-1$; $m_\ell = -\ell \dots +\ell$; $m_s = \pm \frac{1}{2}$

Summary of allowed quantum numbers. Each electron state is uniquely specified by (n, ℓ, m_ℓ, m_s) .

Degeneracy: n^2 orbitals, $2n^2$ states per shell

n^2 = number of distinct spatial orbitals (unique ℓ, m_ℓ combinations) in shell n ; $2n^2$ = total states including both spin orientations ($m_s = \pm 1/2$).

Selection rules $\Delta \ell = \pm 1$, $\Delta m_\ell = 0, \pm 1$

Rules for allowed electric dipole transitions (photon emission or absorption). $\Delta \ell$ = change in orbital quantum number; Δm_ℓ = change in magnetic quantum number. Transitions violating these rules are “forbidden” (occur with negligible probability).

Virial $\langle T \rangle = -E$, $\langle V \rangle = 2E$

Virial theorem for the Coulomb ($1/r$) potential. $\langle T \rangle$ = time-averaged kinetic energy [eV]; $\langle V \rangle$ = time-averaged potential energy [eV]; E = total energy of the state [eV]. For the ground state: $E = -13.6$ eV, so $\langle T \rangle = +13.6$ eV and $\langle V \rangle = -27.2$ eV.

Ionization $I = -E_n = 13.6/n^2$ eV

Ionization energy. I = minimum energy needed to completely remove the electron from level n to $r = \infty$ ($E = 0$) [eV]; E_n = energy of the bound state (negative).

Bohr radius $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 0.0529$ nm

Bohr radius: characteristic length scale of the hydrogen atom. a_0 = most probable distance between electron and nucleus in the ground state [m]; ϵ_0 = permittivity of free space; $\hbar$ = reduced Planck's constant; m_e = electron mass; e = elementary charge.

Interatomic Bonding

$$E(r) = -\frac{A}{r} + \frac{B}{r^m}$$

General interatomic pair potential. $E(r)$ = potential energy between two atoms as a function of their separation r [J or eV]; r = interatomic distance [m]; $-A/r$ = long-range attractive term (Coulombic, van der Waals, etc.); $+B/r^m$ = short-range repulsive term (Pauli exclusion, electron cloud overlap); A = attractive constant [eV m]; B = repulsive constant [eV m^{*m*}]; $m \approx 6$ – 12 = repulsive exponent (always $m > 1$). Equilibrium occurs at the minimum of $E(r)$.

Ionic $A = \frac{Me^2}{4\pi\epsilon_0}$

Attractive constant for ionic bonding. M = Madelung constant (dimensionless), accounts for the summed Coulomb interactions with all ions in the crystal lattice (geometry-dependent); e = elementary charge; ϵ_0 = permittivity of free space.

$$E_{\text{lattice}}(r) = -\frac{Me^2}{4\pi\epsilon_0 r} + \frac{B}{r^m}$$

Total lattice energy per ion pair in an ionic crystal. Same variables as above; this is the pair potential with the Madelung-corrected attractive term that accounts for the entire crystal.

$$E_{\text{bond}} = \frac{A}{r_0} \left(1 - \frac{1}{m} \right)$$

Cohesive (binding) energy at equilibrium. E_{bond} = depth of the potential energy minimum [eV], the energy required to separate the ion pair to infinity; A = attractive constant [eV m]; r_0 = equilibrium bond distance [m] where $dE/dr = 0$; m = repulsive exponent.

Equilibrium $\left. \frac{dE}{dr} \right|_{r_0} = 0$

$$r_0 = \left(\frac{mB \cdot 4\pi\epsilon_0}{e^2 M} \right)^{1/(m-1)}$$

Equilibrium separation. r_0 = bond length where attractive and repulsive forces balance [m]; found by setting the derivative of $E(r)$ to zero and solving. m = repulsive exponent; B = repulsive constant; ϵ_0 = permittivity of free space; e = elementary charge; M = Madelung constant.

Crystal Structures

	atoms	CN	APF	$r(a)$
SC	1	6	0.52	$a/2$
BCC	2	8	0.68	$a\sqrt{3}/4$
FCC	4	12	0.74	$a\sqrt{2}/4$
DC	8	4	0.34	$a\sqrt{3}/8$

Crystal structure properties. SC = simple cubic; BCC = body-centered cubic; FCC = face-centered cubic; DC = diamond cubic (e.g., Si, Ge).

“atoms” = number of atoms per conventional unit cell; CN = coordination number (number of nearest neighbors per atom); APF = atomic packing fraction (volume of atoms / volume of cell); $r(a)$ = relationship between atomic radius r and lattice constant a [m], derived from geometry of touching spheres along the close-packed direction.

Density $\rho = \frac{n_c M_a}{N_A a^3}$

Mass density of a crystal. ρ = density [kg/m³]; n_c = number of atoms per unit cell (from table above); M_a = molar mass (atomic weight) [kg/mol]; N_A = Avogadro's number [mol⁻¹]; a = lattice constant (side length of the cubic unit cell) [m]. The denominator $N_A \cdot a^3$ converts from per-cell to per-volume.

NaCl $M = 1.748$ **CsCl** $M = 1.763$

Madelung constants for common ionic crystal structures. M = dimensionless sum over the crystal lattice accounting for alternating signs of ionic charges and their distances.

Surface $\sigma_{100} = \frac{2}{a^2}$, $\sigma_{110} = \frac{2\sqrt{2}}{a^2}$, $\sigma_{111} = \frac{4}{\sqrt{3}a^2}$

Planar atomic density for diamond cubic (Si). σ_{hkl} = number of atoms per unit area on the (hkl) crystal plane [atoms/m²]; a = lattice constant [m]; (hkl) = Miller indices specifying the crystal plane orientation.

X-ray Diffraction

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Interplanar spacing for cubic crystals. d_{hkl} = perpendicular distance between adjacent (hkl) lattice planes [m]; a = lattice constant [m]; h, k, l = Miller indices (integers specifying the plane family orientation).

$$2d \sin \theta = n\lambda \quad (\text{Bragg's law})$$

Condition for constructive X-ray diffraction from crystal planes. d = interplanar spacing [m]; θ = Bragg angle, the angle between the incident X-ray beam and the crystal plane (not the surface normal) [rad or degrees]; $n = 1, 2, 3, \dots$ = diffraction order (integer); λ = X-ray wavelength [m].

Systematic absences:

BCC: $h+k+l$ = even FCC: h, k, l all odd or all even

Extinction rules: certain reflections are missing due to destructive interference from basis atoms within the unit cell. BCC allows diffraction only when $h+k+l$ is even; FCC allows diffraction only when h, k, l are all odd or all even.

Crystal Defects

$$n_v = N \exp\left(-\frac{E_v}{k_B T}\right)$$

Equilibrium vacancy concentration (Arrhenius relation). n_v = number of vacancies per unit volume [m⁻³]; N = total number of atomic sites per unit volume [m⁻³]; E_v = vacancy formation energy (energy cost to create one vacancy) [J or eV]; k_B = Boltzmann constant; T = absolute temperature [K]. Vacancy concentration increases exponentially with temperature.

Stoichiometry

$$w_A = \frac{x M_A}{x M_A + y M_B}, \quad x_i = \frac{n_i}{\sum_j n_j}$$

Composition measures. w_A = weight fraction of element A in compound $A_x B_y$ (dimensionless, $0 \leq w \leq 1$); x, y = number of atoms of A and B per formula unit; M_A, M_B = molar masses of elements A and B [g/mol]; x_i = mole fraction of component i (dimensionless); n_i = number of moles of component i [mol]; $\sum_j n_j$ = total moles of all components.

Band Theory

$$E_g = E_c - E_v$$

Band gap energy. E_g = band gap [eV], the energy range with no allowed electron states in a semiconductor or insulator; E_c = energy of the conduction band edge (bottom of conduction band) [eV]; E_v = energy of the valence band edge (top of valence band) [eV].

Photoemission Metal: $hf > \Phi$, $KE = hf - \Phi$

Semiconductor: $hf > E_g + \chi$

Conditions for photoemission. In metals: a photon of energy hf must exceed the work function Φ (energy from Fermi level to vacuum). In semiconductors: hf must exceed $E_g + \chi$ to free an electron from the valence band to vacuum. χ = electron affinity, the energy from the conduction band edge E_c to the vacuum level E_{vac} [eV]; Φ = work function, the energy from the Fermi level E_F to the vacuum level [eV]; KE = kinetic energy of the emitted electron.

E-k Dispersion & Effective Mass

$$E = \frac{\hbar^2 k^2}{2m_e^*}$$

Parabolic band approximation near a band extremum. E = electron energy measured from the band edge [J or eV]; $\hbar$ = reduced Planck's constant; k = crystal wavevector (related to the electron's crystal momentum $p = \hbar k$) [m^{-1}]; m_e^* = effective mass of the electron in the crystal [kg]. The effective mass accounts for the periodic potential of the lattice, so the electron behaves as a free particle with mass m_e^* instead of m_e .

$$\frac{1}{m_e^*} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2}$$

Effective mass from band curvature. m_e^* = effective mass [kg]; $\hbar$ = reduced Planck's constant; $d^2 E/dk^2$ = second derivative of the energy dispersion $E(k)$ with respect to wavevector [J m^2]. A sharply curved band (large $d^2 E/dk^2$) gives a small effective mass (light carriers). Negative curvature gives negative effective mass, which defines holes in the valence band.

Density of States

$$g(E) = 8\pi\sqrt{2} \left(\frac{m_e}{\hbar^2} \right)^{3/2} E^{1/2}$$

Density of states for 3D free electrons. $g(E)$ = number of available quantum states per unit energy per unit volume [states/(eV m^3) or $\text{J}^{-1} \text{m}^{-3}$]; m_e = electron mass (or m_e^* in a crystal); $\hbar$ = Planck's constant; E = electron energy measured from the band bottom [J or eV]. The $\sqrt{E}$ dependence means more states are available at higher energies.

$$S_v(E') = \frac{\pi(8m_e E')^{3/2}}{3\hbar^3}$$

Cumulative (integrated) density of states. $S_v(E')$ = total number of states per unit volume with energy up to E' [states/ m^3]; m_e = electron mass; E' = upper energy limit [J]; $\hbar$ = Planck's constant.

Fermi-Dirac Statistics

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}$$

Fermi-Dirac distribution: probability that a quantum state at energy E is occupied by an electron. $f(E)$ = occupation probability (dimensionless, $0 \leq f \leq 1$); E = energy of the state [eV]; E_F = Fermi energy, the energy at which $f = 1/2$ [eV]; k_B = Boltzmann constant; T = absolute temperature [K]. At $T = 0$, f is a step function: $f = 1$ for $E < E_F$, $f = 0$ for $E > E_F$.

Boltzmann approx. $f \approx e^{-(E-E_F)/(k_B T)}$ for $E - E_F \gg k_B T$

Classical (Boltzmann) approximation to Fermi-Dirac, valid when the state energy is far above the Fermi level. Same variables as above.

Boltzmann ratio $\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B T}\right)$

Ratio of populations in two energy levels at thermal equilibrium. N_1, N_2 = number of particles occupying states at energies E_1 and E_2 respectively; $E_2 - E_1$ = energy difference between the two states [J or eV]; k_B = Boltzmann constant; T = temperature [K].

$$E_{F0} = \frac{\hbar^2}{8m_e} \left(\frac{3n}{\pi} \right)^{2/3}$$

Fermi energy at absolute zero ($T = 0$). E_{F0} = Fermi energy at $T = 0$ [J or eV]; $\hbar$ = Planck's constant; m_e = electron mass; n = free electron concentration (number density of conduction electrons) [m^{-3}]. Higher electron density gives higher Fermi energy.

$$E_F(T) = E_{F0} \left[1 - \frac{\pi^2}{12} \left(\frac{k_B T}{E_{F0}} \right)^2 \right]$$

Temperature-corrected Fermi energy. $E_F(T)$ = Fermi energy at temperature T [eV]; E_{F0} = Fermi energy at $T = 0$; k_B = Boltzmann constant; T = temperature [K]. The correction is small for metals at room temperature since $k_B T \ll E_{F0}$.

$$\bar{E} = \frac{3}{5} E_{F0}$$

Average energy per electron at $T = 0$. $\bar{E}$ = mean kinetic energy of all free electrons at absolute zero [eV]; E_{F0} = Fermi energy at $T = 0$. Even at absolute zero, electrons have significant kinetic energy due to the Pauli exclusion principle.

$$\langle E \rangle(T) = \frac{3}{5} E_{F0} \left[1 + \frac{5\pi^2}{12} \left(\frac{k_B T}{E_{F0}} \right)^2 \right]$$

Temperature-corrected average electron energy. $\langle E \rangle(T)$ = mean energy per electron at temperature T [eV]; other variables as above.

Fermi vel. $v_F = \sqrt{2E_F/m_e}$

Fermi velocity: speed of electrons at the Fermi energy. v_F = Fermi velocity [m/s]; E_F = Fermi energy [J]; m_e = electron mass. Derived from $\frac{1}{2} m_e v_F^2 = E_F$. For metals, typically $v_F \sim 10^6$ m/s.

Thermal vel. $v_{rms} = \sqrt{3k_B T/m_e}$

Root-mean-square thermal velocity (classical). v_{rms} = rms speed [m/s]; k_B = Boltzmann constant; T = temperature [K]; m_e = electron mass. For metals at room temperature, $v_{rms} \ll v_F$.

Inverse $n = \frac{\pi}{3} \left(\frac{8m_e E_{F0}}{\hbar^2} \right)^{3/2}$

Electron concentration from Fermi energy (inverted E_{F0} formula). n = free electron concentration [m^{-3}]; m_e = electron mass; E_{F0} = Fermi energy at $T = 0$ [J]; $\hbar$ = Planck's constant.

2D DOS $g_{2D} = \frac{4\pi m_e}{\hbar^2}$

Density of states for a 2D electron gas. g_{2D} = number of states per unit energy per unit area [states/(eV m^2)]; m_e = electron mass (or effective mass m_e^*); $\hbar$ = Planck's constant. Unlike 3D, the 2D DOS is constant (independent of energy).

Drude Model & Conduction

$$v_F = \sqrt{\frac{2E_F}{m_e}}, \quad l = v_F \tau$$

Fermi velocity and mean free path. v_F = Fermi velocity (speed of conduction electrons at the Fermi energy) [m/s]; E_F = Fermi energy [J]; m_e = electron mass; l = mean free path (average distance an electron travels between scattering events) [m]; τ = mean free time (average time between collisions) [s].

$$v_{dx} = \frac{eE_x}{m_e} \tau = \mu E_x$$

Drift velocity in an applied electric field. v_{dx} = drift velocity, the net average velocity of electrons along the field direction [m/s]; e = elementary charge; E_x = applied electric field in the x -direction [V/m]; m_e

= electron mass; τ = mean free time [s]; μ = electron drift mobility [$\text{m}^2/(\text{V s})$].

$$\mu = \frac{e\tau}{m_e}$$

Electron drift mobility. μ = mobility [$\text{m}^2/(\text{V s})$], a measure of how easily electrons move through the material under an electric field; e = elementary charge; τ = mean free time between scattering events [s]; m_e = electron mass (or m_e^* in semiconductors).

$$J_x = en v_{dx} = \sigma E_x$$

Current density (Ohm's law in microscopic form). J_x = current density in the x -direction [A/m^2]; e = elementary charge; n = free electron concentration (number of conduction electrons per unit volume) [m^{-3}]; v_{dx} = drift velocity [m/s]; σ = electrical conductivity [S/m]; E_x = applied electric field [V/m].

$$\sigma = \frac{ne^2\tau}{m_e} = ne\mu$$

Electrical conductivity (Drude formula). σ = conductivity [S/m], inverse of resistivity; n = free electron concentration [m^{-3}]; e = elementary charge; τ = mean free time [s]; m_e = electron mass; μ = electron mobility [$\text{m}^2/(\text{V s})$]. Higher electron density or longer time between collisions gives higher conductivity.

$$R = \frac{\rho L}{A}, \quad \rho = \frac{1}{\sigma}$$

Resistance and resistivity. R = electrical resistance of a conductor [Ω]; ρ = resistivity [$\Omega \text{ m}$], an intrinsic material property (inverse of conductivity σ); L = length of the conductor along the current flow direction [m]; A = cross-sectional area perpendicular to current flow [m^2].

Band model $\sigma = \frac{1}{3}e^2v_F^2\tau g(E_F)$

Quantum (band theory) conductivity. σ = conductivity [S/m]; e = elementary charge; v_F = Fermi velocity [m/s]; τ = mean free time at the Fermi energy [s]; $g(E_F)$ = density of states evaluated at the Fermi energy [states/(J m^3)]. Only electrons near E_F participate in conduction; this formula accounts for that quantum mechanical fact.

Electron conc. $n = \frac{Z \cdot d \cdot N_A}{M_{at}}$

Free electron concentration from material properties. n = conduction electron density [m^{-3}]; Z = valence (number of conduction electrons contributed per atom); d = mass density of the material [kg/m^3]; N_A = Avogadro's number; M_{at} = molar (atomic) mass [kg/mol].

Mean free time $\tau = \frac{m_e}{ne^2\rho}$

Mean free time extracted from measured resistivity. τ = average time between electron scattering events [s]; m_e = electron mass; n = electron concentration [m^{-3}]; e = elementary charge; ρ = measured resistivity [$\Omega \text{ m}$]. Derived from $\rho = 1/\sigma = m_e/(ne^2\tau)$.

Resistivity Temperature Dependence

$$\rho(T) \approx \rho_0[1 + \alpha_0(T - T_0)]$$

Linear approximation for resistivity vs. temperature. $\rho(T)$ = resistivity at temperature T [$\Omega \text{ m}$]; ρ_0 = resistivity at reference temperature T_0 [$\Omega \text{ m}$]; α_0 = temperature coefficient of resistivity at T_0 [K^{-1}]; T = temperature [K]; T_0 = reference temperature [K]. Valid for small temperature changes near T_0 .

Matthiessen $\rho = \rho_T + \rho_I$

Matthiessen's rule: resistivity contributions are additive. ρ = total resistivity [$\Omega \text{ m}$]; ρ_T = contribution from thermal vibrations (phonon scattering), which increases with T ; ρ_I = contribution from impurities and defects (temperature-independent). Each scattering mechanism contributes independently.

Thermoelectrics

$$S = \frac{dV}{dT} \quad (\text{Seebeck coeff.})$$

Seebeck coefficient (thermopower). S = Seebeck coefficient [V/K]; dV = open-circuit voltage developed across a material [V]; dT = temperature difference causing that voltage [K]. $S > 0$ for p-type, $S < 0$ for n-type conductors.

$$V_{AB} = \int_{T_0}^T (S_A - S_B) dT$$

Thermocouple voltage. V_{AB} = voltage measured between the two ends of a thermocouple [V]; S_A, S_B = Seebeck coefficients of materials A and B [V/K]; T_0 = cold junction temperature [K]; T = hot junction temperature [K].

Contact potential $\Delta V = \frac{\Phi_1 - \Phi_2}{e}$

Contact (Volta) potential at a metal-metal junction. ΔV = built-in voltage at the junction [V]; Φ_1, Φ_2 = work functions of materials 1 and 2 [J or eV]; e = elementary charge. Electrons flow from the lower work function material to the higher until Fermi levels align.

Maxwell's Equations

$$\begin{aligned} \nabla \cdot \vec{D} &= \rho_{\text{free}} & \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} & \nabla \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned}$$

Maxwell's equations governing all classical electromagnetism. $\nabla \cdot$ = divergence operator; $\nabla \times$ = curl operator; $\vec{D}$ = electric displacement field [C/m^2]; ρ_{free} = free charge density [C/m^3]; $\vec{B}$ = magnetic flux density [T]; $\vec{E}$ = electric field [V/m]; $\vec{H}$ = magnetic field intensity [A/m]; $\vec{J}$ = free current density [A/m^2]; $\partial/\partial t$ = partial time derivative [s^{-1}]. First eq.: charges produce electric fields. Second: no magnetic monopoles. Third (Faraday): changing $\vec{B}$ induces $\vec{E}$. Fourth (Ampère-Maxwell): currents and changing $\vec{D}$ produce $\vec{H}$.

EM Waves & Optics

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Electromagnetic wave equation in a linear, lossless medium. ∇^2 = Laplacian operator; $\mathbf{E}$ = electric field vector [V/m]; c = phase velocity of light in the medium [m/s]; t = time [s]. Derived from Maxwell's equations in the absence of free charges and currents.

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}, \quad n = \sqrt{\epsilon_r}, \quad c = \frac{c_0}{n}$$

Speed of light and refractive index. c_0 = speed of light in vacuum [m/s]; ϵ_0 = permittivity of free space; μ_0 = permeability of free space; n = refractive index of the medium (dimensionless, $n \geq 1$ for normal materials); ϵ_r = relative permittivity (dielectric constant) of the medium; c = phase velocity of light in the medium [m/s].

Intensity $I = \frac{1}{2}c\epsilon_0 E_0^2$

Time-averaged intensity (irradiance) of a plane electromagnetic wave. I = power per unit area [W/m^2]; c = speed of light in the medium [m/s]; ϵ_0 = permittivity of free space; E_0 = peak (amplitude) of the electric field [V/m]. The factor 1/2 comes from time-averaging $\cos^2$.

Sellmeier $n^2(\lambda) = 1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i}$

Sellmeier dispersion equation: empirical model for wavelength-dependent refractive index. $n(\lambda)$ = refractive index at wavelength λ ; λ = free-space wavelength [m]; B_i = dimensionless strength coefficients for each resonance; C_i = resonance wavelength parameters [m^2] (related to absorption frequencies of the material). Material-specific; typically 2-3 terms.

$$v_g = \frac{c}{N_g}, \quad N_g = n - \lambda \frac{dn}{d\lambda}$$

Group velocity and group index. v_g = group velocity, the speed at which a wave packet (pulse) propagates [m/s]; c = speed of light in vacuum; N_g = group index (dimensionless); n = phase refractive index; λ = wavelength [m]; $dn/d\lambda$ = dispersion of the refractive index [m⁻¹]. In normal dispersion ($dn/d\lambda < 0$), $N_g > n$ so $v_g < c/n$.

Plane wave $E(z, t) = E_0 \cos(kz - \omega t + \phi_0)$

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad v = f\lambda = \frac{\omega}{k}$$

Monochromatic plane wave propagating in the $+z$ direction. $E(z, t)$ = electric field at position z and time t [V/m]; E_0 = amplitude (peak value) [V/m]; k = wavenumber [m⁻¹], the spatial frequency; z = position along propagation direction [m]; ω = angular frequency [rad/s]; t = time [s]; ϕ_0 = initial phase [rad]; λ = wavelength [m]; f = frequency [Hz]; v = phase velocity [m/s].

Snell's Law & Fresnel

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

Snell's law of refraction. n_1 = refractive index of the incident medium; θ_i = angle of incidence (measured from the surface normal) [rad or degrees]; n_2 = refractive index of the transmitted medium; θ_t = angle of transmission (refraction) [rad or degrees].

$$R_s = \left(\frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right)^2$$

Fresnel reflectance for s -polarization (TE). R_s = fraction of incident power reflected for light polarized perpendicular to the plane of incidence (dimensionless, $0 \leq R_s \leq 1$); n_1, n_2 = refractive indices of media 1 and 2; θ_i = angle of incidence; θ_t = angle of transmission (from Snell's law).

$$R_p = \left(\frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \right)^2$$

Fresnel reflectance for p -polarization (TM). R_p = fraction of incident power reflected for light polarized parallel to the plane of incidence. Same variables as R_s . Note the swapped positions of n_1 and n_2 compared to R_s .

Normal inc. $R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$

Reflectance at normal incidence ($\theta_i = 0$). R = fraction of power reflected (dimensionless); n_1, n_2 = refractive indices. At normal incidence $R_s = R_p = R$; polarization does not matter.

$$\tan \theta_B = \frac{n_2}{n_1} \quad (\text{Brewster})$$

Brewster's angle. θ_B = Brewster angle [rad or degrees], the angle of incidence at which $R_p = 0$ (no p -polarized reflection); n_1, n_2 = refractive indices. At θ_B , reflected light is purely s -polarized.

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) \quad (\text{Critical})$$

Critical angle for total internal reflection (TIR). θ_c = critical angle [rad or degrees]; n_1 = refractive index of the denser medium (where light originates); n_2 = refractive index of the less dense medium ($n_2 < n_1$ required). For $\theta_i > \theta_c$, all light is reflected; none is transmitted.

Fiber NA $\text{NA} = \sin \theta_{\text{in,max}} = \sqrt{n_1^2 - n_2^2}$

Numerical aperture of an optical fiber (or waveguide). NA = numerical aperture (dimensionless), characterizes the range of angles over which the fiber accepts light; $\theta_{\text{in,max}}$ = maximum acceptance half-angle at the fiber input [rad or degrees]; n_1 = refractive index of the fiber core; n_2 = refractive index of the cladding ($n_2 < n_1$).

Absorption & Attenuation

$$I(z) = I_0 e^{-\alpha z}$$

Beer-Lambert law: exponential attenuation of light intensity in an absorbing medium. $I(z)$ = intensity (irradiance) at depth z [W/m²]; I_0 =

incident intensity at $z = 0$; α = absorption coefficient [m⁻¹ or Np/m], a measure of how strongly the medium absorbs; z = propagation distance into the medium [m].

$$\alpha = \frac{\pi \varepsilon_r''}{\lambda \sqrt{\varepsilon_r'}}$$

Absorption coefficient from complex permittivity. α = absorption coefficient [m⁻¹]; ε_r'' = imaginary part of relative permittivity (loss factor); λ = free-space wavelength [m]; ε_r' = real part of relative permittivity.

$$\frac{P_{\text{out}}}{P_{\text{in}}} = e^{-2\alpha z}$$

Power attenuation through an absorbing medium. P_{out} = output power [W]; P_{in} = input power [W]; α = absorption coefficient [m⁻¹]; z = propagation distance [m]. The factor of 2 appears because power is proportional to E^2 , so $P \propto I \propto E^2 \propto e^{-2\alpha z}$.

Complex Refractive Index

$$N = n + j\kappa$$

Complex refractive index. N = complex refractive index (not to be confused with number density); n = real part, the ordinary refractive index (determines phase velocity); $j = \sqrt{-1}$ = imaginary unit; κ = extinction coefficient (dimensionless), quantifies absorption (rate of amplitude decay).

Absorption $\alpha = \frac{2\omega\kappa}{c} = \frac{4\pi\kappa}{\lambda}$

Absorption coefficient from extinction coefficient. α = absorption coefficient [m⁻¹]; $\omega = 2\pi f$ = angular frequency [rad/s]; κ = extinction coefficient; c = speed of light in vacuum [m/s]; λ = free-space wavelength [m].

Electric Polarization

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_r \varepsilon_0 \mathbf{E}$$

Constitutive relation for a linear dielectric. $\mathbf{D}$ = electric displacement field [C/m²], accounts for both free and bound charges; ε_0 = permittivity of free space; $\mathbf{E}$ = applied electric field [V/m]; $\mathbf{P}$ = polarization (electric dipole moment per unit volume) [C/m²]; ε_r = relative permittivity (dielectric constant) of the material (dimensionless, $\varepsilon_r \geq 1$).

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}, \quad \varepsilon_r = 1 + \chi_e$$

Polarization and susceptibility. $\mathbf{P}$ = polarization [C/m²]; χ_e = electric susceptibility (dimensionless), measures how easily the material polarizes; ε_0 = permittivity of free space; $\mathbf{E}$ = electric field [V/m]; ε_r = relative permittivity. The "1" in $\varepsilon_r = 1 + \chi_e$ represents the vacuum contribution.

$$\mathbf{P} = N \cdot \mathbf{p}_{\text{avg}}, \quad \mathbf{p} = \alpha_e \mathbf{E}_{\text{loc}}$$

Microscopic origin of polarization. $\mathbf{P}$ = macroscopic polarization [C/m²]; N = number density of polarizable units (atoms, ions, or molecules) [m⁻³]; $\mathbf{p}_{\text{avg}}$ = average induced dipole moment per unit [Cm]; $\mathbf{p}$ = induced dipole moment of a single atom/ion [Cm]; α_e = atomic/ionic polarizability [F m² or C² s²/(kg m³)]; $\mathbf{E}_{\text{loc}}$ = local electric field actually experienced by the atom (differs from the applied field $\mathbf{E}$) [V/m].

$$E_{\text{loc}} = E + \frac{P}{3\varepsilon_0}$$

Lorentz local field at an atomic site in a cubic crystal. E_{loc} = local electric field [V/m]; E = applied (macroscopic) electric field [V/m]; P = polarization [C/m²]; ε_0 = permittivity of free space. The $P/(3\varepsilon_0)$ term is the depolarization correction from surrounding dipoles.

Clausius–Mossotti

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0}$$

Clausius–Mossotti relation: connects the macroscopic dielectric constant to the microscopic polarizability. ε_r = relative permittivity (macroscopic, measurable); N = number density of polarizable units [m^{-3}]; α_e = polarizability per unit [F m^2]; ε_0 = permittivity of free space. Valid for non-polar, cubic-symmetry materials.

$$\text{Rearranged: } \alpha_{\text{total}} = \frac{3\varepsilon_0(\varepsilon_r - 1)}{N(\varepsilon_r + 2)}$$

Inverse form: extract total polarizability from measured ε_r . α_{total} = total polarizability per formula unit [F m^2]. All other variables as above.

Polarizability Mechanisms

$$\alpha_{\text{total}} = \alpha_e + \alpha_i + \alpha_d$$

Total polarizability has three contributions. α_{total} = total polarizability [F m^2]; α_e = electronic polarizability (displacement of electron cloud relative to nucleus; active at all frequencies including optical); α_i = ionic polarizability (displacement of cations relative to anions; active up to infrared frequencies); α_d = dipolar (orientational) polarizability (rotation of permanent dipoles to align with field; active only at low frequencies, below microwave).

$$\alpha_e = 4\pi\varepsilon_0 R^3 \quad (\text{electronic})$$

Electronic polarizability of a spherical atom/ion. α_e = electronic polarizability [F m^2]; ε_0 = permittivity of free space; R = radius of the atom or ion [m]. Larger atoms/ions are more polarizable because the electron cloud is more loosely bound.

$$\alpha_d = \frac{p^2}{3k_B T} \quad (\text{orientational})$$

Orientational (dipolar) polarizability in the weak-field, high-temperature limit. α_d = dipolar polarizability [F m^2]; p = magnitude of the permanent electric dipole moment of the molecule [C m]; k_B = Boltzmann constant; T = absolute temperature [K]. Thermal agitation opposes alignment, so α_d decreases with increasing T .

$$\text{Langevin } L(x) = \coth(x) - \frac{1}{x}, \quad x = \frac{pE}{kT}$$

Langevin function: exact expression for orientational polarization (valid at any field strength). $L(x)$ = Langevin function (dimensionless, approaches 1 for $x \rightarrow \infty$); x = ratio of dipole alignment energy to thermal energy; p = permanent dipole moment [C m]; E = electric field [V/m]; $k = k_B$ = Boltzmann constant; T = temperature [K]. In the weak-field limit ($x \ll 1$), $L(x) \approx x/3$, which yields $\alpha_d = p^2/(3k_B T)$.

Optical Permittivity

$$\alpha_{\text{opt}} = \alpha_e(\text{cation}) + \alpha_e(\text{anion})$$

$$\frac{\varepsilon_{\text{rop}} - 1}{\varepsilon_{\text{rop}} + 2} = \frac{N\alpha_{\text{opt}}}{3\varepsilon_0}$$

Clausius–Mossotti at optical frequencies. α_{opt} = total optical polarizability per ion pair [F m^2], the sum of electronic polarizabilities of the cation and anion (only α_e contributes at optical frequencies; α_i and α_d are too slow to follow); ε_{rop} = optical (high-frequency) relative permittivity; N = number density of ion pairs [m^{-3}]; ε_0 = permittivity of free space.

Debye Relaxation

$$\varepsilon_r(\omega) = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} - \varepsilon_{r\infty}}{1 + j\omega\tau}$$

Debye relaxation model: frequency-dependent complex permittivity for a single relaxation process. $\varepsilon_r(\omega)$ = complex relative permittivity at angular frequency ω ; ε_{rs} = static (DC, $\omega = 0$) relative permittivity; $\varepsilon_{r\infty}$ = high-frequency ($\omega \rightarrow \infty$) relative permittivity; $j = \sqrt{-1}$; $\omega = 2\pi f$ = angular frequency [rad/s]; τ = Debye relaxation time [s], the characteristic time for dipoles to respond to a changing field.

$$\varepsilon'_r = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} - \varepsilon_{r\infty}}{1 + \omega^2\tau^2}, \quad \varepsilon''_r = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})\omega\tau}{1 + \omega^2\tau^2}$$

Real and imaginary parts of the Debye permittivity. $\varepsilon'_r(\omega)$ = real part (storage component), represents energy stored per cycle; $\varepsilon''_r(\omega)$ = imaginary part (loss component), represents energy dissipated per cycle. ε'_r decreases monotonically from ε_{rs} to $\varepsilon_{r\infty}$ as ω increases. ε''_r peaks at $\omega\tau = 1$.

$$\tan \delta = \frac{\varepsilon''_r}{\varepsilon'_r}$$

Loss tangent (dissipation factor). $\tan \delta$ = ratio of energy dissipated to energy stored per cycle (dimensionless); δ = loss angle [rad]; ε''_r = imaginary part of permittivity (loss); ε'_r = real part (storage). Low $\tan \delta$ is desirable for capacitor dielectrics.

Peak of ε''_r occurs at $\omega\tau = 1$: $\varepsilon''_{r,\text{peak}} = (\varepsilon_{rs} - \varepsilon_{r\infty})/2$

$$\text{Complex } \varepsilon_r(\omega) = \varepsilon'_r(\omega) - j\varepsilon''_r(\omega)$$

General complex permittivity notation. The negative sign convention ensures that $\varepsilon''_r > 0$ corresponds to loss (energy absorption). ε'_r and ε''_r are both real, positive quantities.

$$\text{Complex } k \quad \tilde{k} = \frac{\omega}{c}\sqrt{\varepsilon_r} = k' + ik''$$

Complex wavevector in a lossy medium. $\tilde{k}$ = complex wavenumber [m^{-1}]; ω = angular frequency [rad/s]; c = speed of light in vacuum [m/s]; ε_r = complex relative permittivity; k' = real part (propagation constant, determines phase velocity $v_p = \omega/k'$) [m^{-1}]; k'' = imaginary part (attenuation constant, determines amplitude decay) [m^{-1}].

Wave in Lossy Medium

$$E(z) = E_0 e^{-k''z} e^{i(k'z - \omega t)}$$

Electric field of a plane wave in an absorbing (lossy) medium. $E(z)$ = electric field at position z and time t [V/m]; E_0 = initial amplitude [V/m]; $e^{-k''z}$ = exponential decay envelope (attenuation); k'' = imaginary part of the complex wavevector [m^{-1}]; z = propagation distance [m]; k' = real part of the wavevector [m^{-1}]; ω = angular frequency [rad/s]; t = time [s]. The wave both propagates (via k') and decays (via k'').

Gauss's Law for D

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}}, \quad \oint \mathbf{D} \cdot d\mathbf{A} = Q_{\text{free,enclosed}}$$

Gauss's law in differential and integral form. $\nabla \cdot \mathbf{D}$ = divergence of the displacement field; ρ_{free} = volume density of free charges [C/m^3]; $\oint \mathbf{D} \cdot d\mathbf{A}$ = flux of $\mathbf{D}$ through a closed surface; $d\mathbf{A}$ = outward-pointing area element [m^2]; $Q_{\text{free,enclosed}}$ = total free charge enclosed by the surface [C]. Bound (polarization) charges are absorbed into $\mathbf{D}$.

$$\text{Boundary } D_{n1} - D_{n2} = \sigma_f$$

Boundary condition for the normal component of $\mathbf{D}$ at an interface. D_{n1} = normal component of $\mathbf{D}$ in medium 1 [C/m^2]; D_{n2} = normal component in medium 2; σ_f = free surface charge density at the interface [C/m^2]. If no free surface charge ($\sigma_f = 0$), then $D_{n1} = D_{n2}$.

Capacitors & Energy

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Capacitance of a parallel-plate capacitor. C = capacitance [F]; ε_r = relative permittivity of the dielectric between plates; ε_0 = permittivity of free space; A = area of each plate [m^2]; d = separation between plates [m].

$$u = \frac{1}{2}\varepsilon_r \varepsilon_0 E^2 = \frac{1}{2}\mathbf{D} \cdot \mathbf{E}$$

Energy density stored in an electric field. u = energy per unit volume [J/m^3]; ε_r = relative permittivity; ε_0 = permittivity of free space; E = electric field magnitude [V/m]; $\mathbf{D}$ = displacement field [C/m^2]. Total stored energy: $U = \frac{1}{2}CV^2$, where V = voltage across the capacitor [V].

$$\text{Parallel } C_{\text{tot}} = C_1 + C_2 + \dots \quad \text{Series } 1/C_{\text{tot}} = 1/C_1 + 1/C_2 + \dots$$

Capacitors in parallel and in series. C_{tot} = total equivalent capacitance [F]; $C_1, C_2, \dots$ = individual capacitances. Parallel: same voltage, charges add. Series: same charge, voltages add.

$$\text{Multilayer } C = \frac{\epsilon_0 A}{d_1/\epsilon_{r1} + d_2/\epsilon_{r2}}$$

Capacitance with two dielectric layers in series. C = total capacitance [F]; ϵ_0 = permittivity of free space; A = plate area [m²]; d_1, d_2 = thicknesses of dielectric layers 1 and 2 [m]; $\epsilon_{r1}, \epsilon_{r2}$ = relative permittivities of the two layers. Each layer acts as a capacitor in series.

Piezoelectricity

$$P = d \cdot T \text{ (direct), } S = d \cdot E \text{ (converse)}$$

Piezoelectric effect. Direct effect: mechanical stress produces polarization. Converse effect: electric field produces strain. P = induced polarization [C/m²]; d = piezoelectric coefficient [C/N or m/V]; T = applied mechanical stress [N/m² = Pa]; S = induced mechanical strain (fractional deformation, dimensionless); E = applied electric field [V/m]. Note: d has the same numerical value in both equations.

$$f = \frac{v}{2L} \text{ (resonance), } Q = \frac{f_{\text{res}}}{\Delta f}$$

Resonance of a piezoelectric crystal. f = fundamental resonant frequency [Hz]; v = speed of sound (acoustic velocity) in the crystal [m/s]; L = thickness of the crystal in the vibration direction [m]; the crystal resonates when L equals half an acoustic wavelength. Q = quality factor (dimensionless), measures the sharpness of the resonance; f_{res} = resonant frequency [Hz]; Δf = full width at half maximum (FWHM) of the resonance peak [Hz]. Higher Q means lower energy loss per cycle.

Standing Waves & Interference

$$E = 2E_0 \cos(kz) \cos(\omega t)$$

Standing wave formed by superposition of two counter-propagating plane waves of equal amplitude. E = electric field [V/m]; E_0 = amplitude of each individual wave [V/m]; $k = 2\pi/\lambda$ = wavenumber [m⁻¹]; z = position [m]; $\omega = 2\pi f$ = angular frequency [rad/s]; t = time [s]. Nodes (zero amplitude) are spaced $\lambda/2$ apart; antinodes (maximum amplitude $2E_0$) are halfway between nodes.

$$\text{Double slit } \Delta = d \sin \theta$$

Path difference in Young's double-slit experiment. Δ = path length difference between waves from the two slits arriving at angle θ [m]; d = slit separation (distance between the two slits) [m]; θ = observation angle from the central axis [rad or degrees]. Constructive interference: $\Delta = n\lambda$ ($n = 0, 1, 2, \dots$). Destructive interference: $\Delta = (n + 1/2)\lambda$.

Superposition (Infinite Well)

$$\Psi(x, t) = c_1 \varphi_1 e^{-iE_1 t/\hbar} + c_2 \varphi_2 e^{-iE_2 t/\hbar}$$

Time-dependent superposition of two stationary states. $\Psi(x, t)$ = total wavefunction; c_1, c_2 = expansion coefficients (probability amplitudes, dimensionless, with $|c_1|^2 + |c_2|^2 = 1$); φ_1, φ_2 = spatial wavefunctions of states 1 and 2 (e.g., $\sqrt{2/a} \sin(n\pi x/a)$); E_1, E_2 = energies of states 1 and 2 [J]; $\hbar$ = reduced Planck's constant; t = time [s]; $i = \sqrt{-1}$. The $e^{-iEt/\hbar}$ factors are the time-evolution phases.

$$|\Psi|^2 = c_1^2 \varphi_1^2 + c_2^2 \varphi_2^2 + 2c_1 c_2 \varphi_1 \varphi_2 \cos\left(\frac{(E_2 - E_1)t}{\hbar}\right)$$

Probability density of the superposition state (assuming real c_1, c_2). The first two terms are the static contributions from each state; the third (cross) term oscillates in time. Oscillation angular frequency: $\omega_{21} = (E_2 - E_1)/\hbar$ [rad/s]. The probability density "sloshes" back and forth in the well at this frequency.

Material Properties

Material	a (nm)	E_g (eV)
Si	0.543	1.12
Ge	0.566	0.66
GaAs	0.565	1.42
Diamond	0.357	5.5

a = cubic lattice constant [nm]; E_g = band gap energy at room temperature [eV]. Si and Ge are indirect-gap semiconductors (diamond cubic); GaAs is direct-gap (zinc blende).

$$\text{Si: } n_{\text{Si}} = 4.99 \times 10^{22} \text{ cm}^{-3}, \rho = 2.33 \text{ g/cm}^3$$

$$n_{\text{Si}}(\text{refractive}) = 3.48, \quad n_{\text{SiO}_2} = 1.44$$

n_{Si} (first) = atomic number density of silicon [cm⁻³]; ρ = mass density [g/cm³]; n_{Si} (refractive) = refractive index of silicon at ~ 1550 nm; n_{SiO_2} = refractive index of silicon dioxide (glass).

Spectroscopic Notation

$$\ell = 0, 1, 2, 3 \rightarrow s, p, d, f$$

Shell capacity: $2(2\ell + 1)$ electrons per subshell

Mapping of orbital quantum number ℓ to letter labels. $\ell = 0 \rightarrow s$, $\ell = 1 \rightarrow p$, $\ell = 2 \rightarrow d$, $\ell = 3 \rightarrow f$. Each subshell with quantum number ℓ holds up to $2(2\ell + 1)$ electrons: s holds 2, p holds 6, d holds 10, f holds 14. The factor 2 accounts for spin ($m_s = \pm 1/2$); $(2\ell + 1)$ counts the allowed m_ℓ values.

Useful Identities & Conversions

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}, \quad 1 \text{ nm} = 10^{-9} \text{ m}, \quad 1 \text{ \AA} = 0.1 \text{ nm}$$

$$E = hc/\lambda: \text{ at } \lambda = 1240 \text{ nm}, E = 1 \text{ eV}$$

$$\text{Euler } e^{i\theta} = \cos \theta + i \sin \theta$$

Euler's formula. e = base of the natural logarithm (≈ 2.718); $i = \sqrt{-1}$; θ = angle [rad]. Connects complex exponentials to trigonometric functions; fundamental to wave physics.

$$\sinh(x) = (e^x - e^{-x})/2, \quad \cosh(x) = (e^x + e^{-x})/2$$

Hyperbolic sine and cosine. Used in tunneling transmission ($\sinh$) and standing wave analysis ($\cosh$). x = argument (dimensionless in physics applications).

$$\text{Stopping } eV_{\text{stop}} = KE_{\text{max}} = hf - \Phi$$

Stopping potential in the photoelectric effect. V_{stop} = stopping voltage [V], the reverse bias needed to halt the most energetic photoelectrons; e = elementary charge; KE_{max} = maximum kinetic energy [J or eV]; hf = photon energy; Φ = work function. Measuring V_{stop} vs. f gives h/e as the slope and $-\Phi/e$ as the intercept.

$$\hbar c = 197.3 \text{ eV nm}, \quad m_e c^2 = 0.511 \text{ MeV}$$

Useful constants. $\hbar c$ = product of reduced Planck's constant and speed of light [eV nm]; $m_e c^2$ = electron rest mass energy [MeV].

$$\text{Dispersion } E = \hbar\omega, p = \hbar k$$

Dispersion relations connecting energy/momentum to wave properties. E = energy [J]; $\hbar$ = reduced Planck's constant; ω = angular frequency [rad/s]; p = momentum [kg m/s]; k = wavevector [m⁻¹].

$$\text{Free electron } \psi = Ae^{jkx}, k = \sqrt{2m_e E}/\hbar$$

Free electron (plane wave) wavefunction. ψ = wavefunction; A = normalization constant; $j = \sqrt{-1}$; k = wavevector [m⁻¹]; x = position [m]; m_e = electron mass; E = kinetic energy [J]; $\hbar$ = reduced Planck's constant.

$$R + T = 1 \text{ (reflection + transmission = 1)}$$

Conservation of probability (or power). R = reflection coefficient (probability of reflection); T = transmission coefficient (probability of transmission). All incident flux must be either reflected or transmitted.

$$N = n_c/a^3 \text{ (number density from unit cell)}$$

Number density of atoms/ions. N = number of atoms or ion pairs per unit volume [m⁻³]; n_c = number of atoms (or formula units) per unit cell; a = lattice constant [m]; a^3 = volume of the cubic unit cell [m³].