

Physical Constants

h	$6.626 \times 10^{-34} \text{ J s}$
$\hbar = h/2\pi$	$1.055 \times 10^{-34} \text{ J s}$
hc	1240 eV nm
m_e	$9.109 \times 10^{-31} \text{ kg}$
e	$1.602 \times 10^{-19} \text{ C}$
k_B	$1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$
ε_0	$8.854 \times 10^{-12} \text{ F/m}$
μ_0	$4\pi \times 10^{-7} \text{ N/A}^2$
c_0	$3.0 \times 10^8 \text{ m/s}$
N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$
a_0	0.0529 nm (Bohr radius)
$e^2/(4\pi\varepsilon_0)$	1.44 eV nm (Coulomb shortcut)
$k_B T _{300\text{K}}$	0.02585 eV

Wave-Particle Duality

$$E = hf = \hbar\omega$$

Energy of a photon; f = frequency, $\omega = 2\pi f$

$$p = \frac{h}{\lambda} = \frac{\hbar f}{c}$$

Photon momentum

$$\text{Photoelectric: } KE_{\max} = hf - \Phi$$

Φ = work function (min. energy to extract electron)

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV}}$$

de Broglie wavelength; second form for electron through potential V

$$\text{Wien } \lambda_{\max} T = 2.898 \times 10^{-3} \text{ m K}$$

Peak wavelength of blackbody radiation

$$\text{Photon flux } \dot{N} = \frac{P}{E_{ph}} = \frac{P\lambda}{hc}$$

Photon rate from source of power P

$$\text{Thermal } \lambda \lambda_B = h/\sqrt{3mk_B T}$$

Thermal de Broglie wavelength; quantum regime when $\lambda_B \gtrsim$ interparticle spacing

Schrödinger Equation

$$\text{Momentum op. } \hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\text{Expectation } \langle A \rangle = \int \psi^* \hat{A} \psi dx$$

Mean value of observable A ; $\hat{A}$ is the quantum operator

$$\text{Normalization } \int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

$$\text{TDSE } i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$$

Time-dependent Schrödinger equation

$$\text{Hamiltonian } \hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r})$$

Total energy operator = kinetic + potential

$$\text{TISE } \frac{d^2 \psi}{dx^2} + \frac{2m_e}{\hbar^2} (E - V) \psi = 0$$

Time-independent form; solve for stationary states

Infinite Potential Well

Particle confined to $0 < x < a$, $V = \infty$ outside

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2m_e a^2} = \frac{n^2 \hbar^2}{8m_e a^2}$$

a = well width, $n = 1, 2, 3, \dots$ (quantum number)

$$|\psi_n|^2 = \frac{2}{a} \sin^2\left(\frac{n\pi x}{a}\right)$$

Probability density; peaks at antinodes

$$\Delta E_n = E_{n+1} - E_n = \frac{(2n+1)\hbar^2}{8m_e a^2}$$

$$\text{3D box } E_{n_1 n_2 n_3} = \frac{\hbar^2}{8m_e} \left(\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2} + \frac{n_3^2}{c^2} \right)$$

a, b, c = box dimensions; degeneracy when $a = b = c$

Finite Potential Well

$$k^2 = \frac{2m_e E}{\hbar^2}, \quad \alpha^2 = \frac{2m_e (V_0 - E)}{\hbar^2}$$

k = wavevector inside well, α = decay constant outside

$$\text{Even } \alpha = k \tan\left(\frac{ka}{2}\right)$$

$$\text{Odd } \alpha = -k \cot\left(\frac{ka}{2}\right)$$

Transcendental equations; solve graphically for bound state energies

$$\text{Penetration } \delta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m_e (V_0 - E)}}$$

Depth of wavefunction tail into classically forbidden region

$$\text{Bound states } N \leq \left\lfloor \frac{1}{2} + \frac{a}{\pi} \sqrt{\frac{2m_e V_0}{\hbar^2}} \right\rfloor$$

Quantum Tunneling

$$T = \frac{1}{1 + D \sinh^2(\alpha a)}$$

$$D = \frac{V_0^2}{4E(V_0 - E)}, \quad \alpha = \frac{\sqrt{2m_e (V_0 - E)}}{\hbar}$$

T = transmission probability through barrier of height V_0 , width a

$$\text{Thick barrier } T \approx T_0 e^{-2\alpha a}, \quad T_0 = \frac{16E(V_0 - E)}{V_0^2}$$

Valid when $\alpha a \gg 1$ (opaque barrier)

$$\text{STM current } I \propto V_b e^{-2\alpha d}, \quad \alpha = \frac{\sqrt{2m_e \phi}}{\hbar}$$

Scanning tunneling microscope; d = tip-sample gap, ϕ = avg. work function

Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}, \quad \Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

Fundamental limits on simultaneous measurement precision

Hydrogen Atom

$$V(r) = \frac{-Ze^2}{4\pi\varepsilon_0 r}$$

Coulomb potential; Z = atomic number (1 for H)

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

n = principal quantum number; ground state $n = 1$

$$L = \hbar \sqrt{\ell(\ell+1)}, \quad L_z = m_\ell \hbar$$

Orbital angular momentum; $\ell = 0 \dots n-1$

$$S = \hbar \sqrt{s(s+1)} = \frac{\sqrt{3}}{2} \hbar, \quad S_z = m_s \hbar$$

Spin; $s = 1/2$ for electrons, $m_s = \pm 1/2$

Quantum numbers: $n = 1, 2, \dots$; $\ell = 0 \dots n-1$; $m_\ell = -\ell \dots +\ell$; $m_s = \pm \frac{1}{2}$

Degeneracy: n^2 orbitals, $2n^2$ states per shell

Selection rules $\Delta\ell = \pm 1$, $\Delta m_\ell = 0, \pm 1$

Allowed electric dipole transitions

Virial $\langle T \rangle = -E$, $\langle V \rangle = 2E$

Avg. kinetic and potential energy from total E

Ionization $I = -E_n = 13.6/n^2$ eV

Bohr radius $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 0.0529$ nm

Interatomic Bonding

$$E(r) = -\frac{A}{r} + \frac{B}{r^m}$$

Pair potential: attractive ($-A/r$) + repulsive (B/r^m , $m \approx 6-12$)

Ionic $A = \frac{Me^2}{4\pi\epsilon_0}$

M = Madelung constant (depends on crystal geometry)

$$E_{\text{lattice}}(r) = -\frac{Me^2}{4\pi\epsilon_0 r} + \frac{B}{r^m}$$

$$E_{\text{bond}} = \frac{A}{r_0} \left(1 - \frac{1}{m} \right)$$

Cohesive energy at equilibrium separation r_0

Equilibrium $\left. \frac{dE}{dr} \right|_{r_0} = 0$

$$r_0 = \left(\frac{mB \cdot 4\pi\epsilon_0}{e^2 M} \right)^{1/(m-1)}$$

Crystal Structures

	atoms	CN	APF	$r(a)$
SC	1	6	0.52	$a/2$
BCC	2	8	0.68	$a\sqrt{3}/4$
FCC	4	12	0.74	$a\sqrt{2}/4$
DC	8	4	0.34	$a\sqrt{3}/8$

CN = coordination number, APF = atomic packing fraction

Density $\rho = \frac{n_c M_a}{N_A a^3}$

n_c = atoms/cell, M_a = molar mass, a = lattice constant

NaCl $M = 1.748$ **CsCl** $M = 1.763$

Surface $\sigma_{100} = \frac{2}{a^2}$, $\sigma_{110} = \frac{2\sqrt{2}}{a^2}$, $\sigma_{111} = \frac{4}{\sqrt{3}a^2}$

Planar atomic density for diamond cubic (Si) [atoms/area]

X-ray Diffraction

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Interplanar spacing for cubic crystals

$$2d \sin \theta = n\lambda \quad (\text{Bragg's law})$$

θ = diffraction angle, n = order, λ = X-ray wavelength

Systematic absences:

BCC: $h+k+l$ = even FCC: h, k, l all odd or all even

Crystal Defects

$$n_v = N \exp\left(-\frac{E_v}{k_B T}\right)$$

n_v = vacancy concentration, E_v = vacancy formation energy

Stoichiometry

$$w_A = \frac{x M_A}{x M_A + y M_B}, \quad x_i = \frac{n_i}{\sum_j n_j}$$

w_A = weight fraction, x_i = mole fraction

Band Theory

$$E_g = E_c - E_v$$

E_g = band gap; E_c = conduction band edge; E_v = valence band edge

Photoemission Metal: $hf > \Phi$, $KE = hf - \Phi$

Semiconductor: $hf > E_g + \chi$

χ = electron affinity ($E_{\text{vac}} - E_c$); Φ = work function ($E_{\text{vac}} - E_F$)

E-k Dispersion & Effective Mass

$$E = \frac{\hbar^2 k^2}{2m_e^*}$$

Parabolic approximation near band extrema

$$\frac{1}{m_e^*} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2}$$

Effective mass from curvature of E - k band

Density of States

$$g(E) = 8\pi\sqrt{2} \left(\frac{m_e}{\hbar^2} \right)^{3/2} E^{1/2}$$

States per unit energy per unit volume (3D free electron)

$$S_v(E') = \frac{\pi(8m_e E')^{3/2}}{3\hbar^3}$$

Cumulative states up to energy E'

Fermi-Dirac Statistics

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}$$

Probability that state at energy E is occupied; E_F = Fermi energy

Boltzmann approx. $f \approx e^{-(E-E_F)/(k_B T)}$ for $E - E_F \gg k_B T$

Boltzmann ratio $\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B T}\right)$

$$E_{F0} = \frac{\hbar^2}{8m_e} \left(\frac{3n}{\pi} \right)^{2/3}$$

Fermi energy at $T = 0$; n = free electron concentration

$$E_F(T) = E_{F0} \left[1 - \frac{\pi^2}{12} \left(\frac{k_B T}{E_{F0}} \right)^2 \right]$$

$$\bar{E} = \frac{3}{5} E_{F0}$$

Average electron energy at $T = 0$

$$\langle E \rangle(T) = \frac{3}{5} E_{F0} \left[1 + \frac{5\pi^2}{12} \left(\frac{k_B T}{E_{F0}} \right)^2 \right]$$

Temperature-corrected average energy

Fermi vel. $v_F = \sqrt{2E_F/m_e}$

Thermal vel. $v_{\text{rms}} = \sqrt{3k_B T/m_e}$

Inverse $n = \frac{\pi}{3} \left(\frac{8m_e E_{F0}}{\hbar^2} \right)^{3/2}$

Electron concentration from Fermi energy (inverted E_{F0} formula)

2D DOS $g_{2D} = \frac{4\pi m_e}{\hbar^2}$ (energy-independent, per unit area)

Drude Model & Conduction

$$v_F = \sqrt{\frac{2E_F}{m_e}}, \quad l = v_F \tau$$

v_F = Fermi velocity, l = mean free path, τ = mean free time

$$v_{dx} = \frac{eE_x}{m_e} \tau = \mu E_x$$

v_{dx} = drift velocity in applied field E_x

$$\mu = \frac{e\tau}{m_e}$$

μ = electron mobility [m²/(V s)]

$$J_x = en v_{dx} = \sigma E_x$$

J_x = current density [A/m²]

$$\sigma = \frac{ne^2\tau}{m_e} = ne\mu$$

σ = electrical conductivity [S/m]

$$R = \frac{\rho L}{A}, \quad \rho = \frac{1}{\sigma}$$

R = resistance, ρ = resistivity, L = length, A = cross-section

Band model $\sigma = \frac{1}{3} e^2 v_F^2 \tau g(E_F)$

Quantum conductivity from Fermi surface scattering

$$\text{Electron conc. } n = \frac{Z \cdot d \cdot N_A}{M_{at}}$$

Z = valence electrons, d = mass density, M_{at} = atomic mass

$$\text{Mean free time } \tau = \frac{m_e}{ne^2\rho}$$

Extract τ from measured resistivity

Resistivity Temperature Dependence

$$\rho(T) \approx \rho_0[1 + \alpha_0(T - T_0)]$$

Linear approximation; α_0 = temperature coefficient of resistivity

Matthiessen $\rho = \rho_T + \rho_I$

Total = thermal scattering + impurity scattering (additive)

Thermoelectrics

$$S = \frac{dV}{dT} \quad (\text{Seebeck coeff.})$$

Voltage generated per unit temperature difference [V/K]

$$V_{AB} = \int_{T_0}^T (S_A - S_B) dT$$

Thermocouple voltage between materials A and B

$$\text{Contact potential } \Delta V = \frac{\Phi_1 - \Phi_2}{e}$$

Built-in voltage from work function difference at junction

Maxwell's Equations

$$\nabla \cdot \vec{D} = \rho_{\text{free}} \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

EM Waves & Optics

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

Wave equation in a linear medium

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}, \quad n = \sqrt{\epsilon_r}, \quad c = \frac{c_0}{n}$$

n = refractive index, ϵ_r = relative permittivity

$$\text{Intensity } I = \frac{1}{2} c \epsilon_0 E_0^2$$

Time-averaged power per unit area for plane wave

$$\text{Sellmeier } n^2(\lambda) = 1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i}$$

Empirical dispersion; B_i, C_i = material coefficients

$$v_g = \frac{c}{N_g}, \quad N_g = n - \lambda \frac{dn}{d\lambda}$$

v_g = group velocity, N_g = group index

Plane wave $E(z, t) = E_0 \cos(kz - \omega t + \phi_0)$

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad v = f\lambda = \frac{\omega}{k}$$

Snell's Law & Fresnel

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

θ_i = angle of incidence, θ_t = angle of transmission

$$R_s = \left(\frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right)^2$$

s -polarized reflectance ($\perp$ to plane of incidence)

$$R_p = \left(\frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \right)^2$$

p -polarized reflectance ($\parallel$ to plane of incidence)

$$\text{Normal inc. } R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$\tan \theta_B = \frac{n_2}{n_1} \quad (\text{Brewster})$$

Angle where $R_p = 0$; reflected light is purely s -polarized

$$\theta_c = \arcsin\left(\frac{n_2}{n_1}\right) \quad (\text{Critical})$$

Total internal reflection for $\theta_i > \theta_c$ (requires $n_1 > n_2$)

$$\text{Fiber NA } \text{NA} = \sin \theta_{\text{in,max}} = \sqrt{n_1^2 - n_2^2}$$

Numerical aperture; max acceptance angle for waveguide

Absorption & Attenuation

$$I(z) = I_0 e^{-\alpha z}$$

Beer-Lambert law; α = absorption coefficient [Np/m]

$$\alpha = \frac{\pi \epsilon_r''}{\lambda \sqrt{\epsilon_r'}}$$

$$\frac{P_{\text{out}}}{P_{\text{in}}} = e^{-2\alpha z}$$

Power attenuation uses 2α (intensity $\propto E^2$)

Complex Refractive Index

$$N = n + j\kappa$$

n = refractive index, κ = extinction coefficient

$$\text{Absorption } \alpha = \frac{2\omega\kappa}{c} = \frac{4\pi\kappa}{\lambda}$$

Electric Polarization

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_r \epsilon_0 \mathbf{E}$$

$\mathbf{D}$ = displacement field, $\mathbf{P}$ = polarization

$$\mathbf{P} = \chi_e \epsilon_0 \mathbf{E}, \quad \epsilon_r = 1 + \chi_e$$

χ_e = electric susceptibility $\mathbf{P} = N \cdot \mathbf{p}_{\text{avg}}, \quad \mathbf{p} = \alpha_e \mathbf{E}_{\text{loc}}$

N = number density, α_e = polarizability

$$E_{\text{loc}} = E + \frac{P}{3\epsilon_0}$$

Lorentz local field at atomic site $\neq$ applied field

Clausius–Mossotti

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0}$$

Links macroscopic ε_r to microscopic polarizability α_e

$$\text{Rearranged: } \alpha_{\text{total}} = \frac{3\varepsilon_0(\varepsilon_r - 1)}{N(\varepsilon_r + 2)}$$

Polarizability Mechanisms

$$\alpha_{\text{total}} = \alpha_e + \alpha_i + \alpha_d$$

Electronic + ionic + orientational (dipolar)

$$\alpha_e = 4\pi\varepsilon_0 R^3 \quad (\text{electronic})$$

R = atomic/ionic radius; dominates at all frequencies

$$\alpha_d = \frac{p^2}{3k_B T} \quad (\text{orientational})$$

p = permanent dipole moment; active only at low ω

$$\text{Langevin } L(x) = \coth(x) - \frac{1}{x}, \quad x = \frac{pE}{kT}$$

Exact orientational polarization; α_d is the weak-field limit

Optical Permittivity

$$\alpha_{\text{opt}} = \alpha_e(\text{cation}) + \alpha_e(\text{anion})$$

$$\frac{\varepsilon_{\text{rop}} - 1}{\varepsilon_{\text{rop}} + 2} = \frac{N\alpha_{\text{opt}}}{3\varepsilon_0}$$

At optical frequencies only electronic polarizability contributes

Debye Relaxation

$$\varepsilon_r(\omega) = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} - \varepsilon_{r\infty}}{1 + j\omega\tau}$$

ε_{rs} = static (DC), $\varepsilon_{r\infty}$ = high-freq., τ = relaxation time

$$\varepsilon'_r = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} - \varepsilon_{r\infty}}{1 + \omega^2\tau^2}, \quad \varepsilon''_r = \frac{(\varepsilon_{rs} - \varepsilon_{r\infty})\omega\tau}{1 + \omega^2\tau^2}$$

ε'_r = storage (real), ε''_r = loss (imaginary)

$$\tan \delta = \frac{\varepsilon''_r}{\varepsilon'_r}$$

Loss tangent; ratio of energy dissipated to stored per cycle

Peak of ε''_r at $\omega\tau = 1$: $\varepsilon''_{r,\text{peak}} = (\varepsilon_{rs} - \varepsilon_{r\infty})/2$

$$\text{Complex } \varepsilon \quad \varepsilon_r(\omega) = \varepsilon'_r(\omega) - j\varepsilon''_r(\omega)$$

$$\text{Complex } k \quad \tilde{k} = \frac{\omega}{c}\sqrt{\varepsilon_r} = k' + ik''$$

Wave in Lossy Medium

$$E(z) = E_0 e^{-k''z} e^{i(k'z - \omega t)}$$

k'' attenuates amplitude; k' determines phase velocity

Gauss's Law for D

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}}, \quad \oint \mathbf{D} \cdot d\mathbf{A} = Q_{\text{free,enclosed}}$$

$$\text{Boundary } D_{n1} - D_{n2} = \sigma_f$$

σ_f = free surface charge density at interface

Capacitors & Energy

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

A = plate area, d = plate separation

$$u = \frac{1}{2}\varepsilon_r \varepsilon_0 E^2 = \frac{1}{2}\mathbf{D} \cdot \mathbf{E}$$

Energy density [J/m³]; total $U = \frac{1}{2}CV^2$

$$\text{Parallel } C_{\text{tot}} = C_1 + C_2 + \dots \quad \text{Series } 1/C_{\text{tot}} = 1/C_1 + 1/C_2 + \dots$$

$$\text{Multilayer } C = \frac{\varepsilon_0 A}{d_1/\varepsilon_{r1} + d_2/\varepsilon_{r2}}$$

Two dielectric layers in series

Piezoelectricity

$$P = d \cdot T \quad (\text{direct}), \quad S = d \cdot E \quad (\text{converse})$$

d = piezoelectric coefficient, T = stress, S = strain

$$f = \frac{v}{2L} \quad (\text{resonance}), \quad Q = \frac{f_{\text{res}}}{\Delta f}$$

v = acoustic velocity in crystal, L = thickness, Q = quality factor

Standing Waves & Interference

$$E = 2E_0 \cos(kz) \cos(\omega t)$$

Superposition of counter-propagating waves; antinode spacing $\lambda/2$

$$\text{Double slit } \Delta = d \sin \theta$$

Path difference; constructive $\Delta = n\lambda$, destructive $\Delta = (n + \frac{1}{2})\lambda$

Superposition (Infinite Well)

$$\Psi(x, t) = c_1 \varphi_1 e^{-iE_1 t/\hbar} + c_2 \varphi_2 e^{-iE_2 t/\hbar}$$

$$|\Psi|^2 = c_1^2 \varphi_1^2 + c_2^2 \varphi_2^2 + 2c_1 c_2 \varphi_1 \varphi_2 \cos\left(\frac{(E_2 - E_1)t}{\hbar}\right)$$

Oscillation frequency: $\omega_{21} = (E_2 - E_1)/\hbar$

Material Properties

Material	a (nm)	E_g (eV)
Si	0.543	1.12
Ge	0.566	0.66
GaAs	0.565	1.42
Diamond	0.357	5.5

Si: $n_{\text{Si}} = 4.99 \times 10^{22} \text{ cm}^{-3}$, $\rho = 2.33 \text{ g/cm}^3$

$n_{\text{Si}}(\text{refractive}) = 3.48$, $n_{\text{SiO}_2} = 1.44$

Spectroscopic Notation

$\ell = 0, 1, 2, 3 \rightarrow s, p, d, f$

Shell capacity: $2(2\ell + 1)$ electrons per subshell

Useful Identities & Conversions

1 eV = $1.602 \times 10^{-19} \text{ J}$, 1 nm = 10^{-9} m , 1 Å = 0.1 nm

$E = hc/\lambda$: at $\lambda = 1240 \text{ nm}$, $E = 1 \text{ eV}$

$$\text{Euler } e^{i\theta} = \cos \theta + i \sin \theta$$

$$\sinh(x) = (e^x - e^{-x})/2, \quad \cosh(x) = (e^x + e^{-x})/2$$

$$\text{Stopping } eV_{\text{stop}} = KE_{\text{max}} = hf - \Phi$$

$$\hbar c = 197.3 \text{ eV nm}, \quad m_e c^2 = 0.511 \text{ MeV}$$

$$\text{Dispersion } E = \hbar\omega, \quad p = \hbar k$$

$$\text{Free electron } \psi = Ae^{ikx}, \quad k = \sqrt{2m_e E}/\hbar$$

$$R + T = 1 \quad (\text{reflection} + \text{transmission} = 1)$$

$$N = n_c/a^3 \quad (\text{number density from unit cell})$$